Eq. lineares de 1ª ordem

$$y' + P(x)\,y = Q(x)\,, \quad P(x) \text{ e } Q(x) \text{ dadas}, \quad y(x) = ?$$

Exemplos: 1) $y' + \underbrace{\frac{1}{x}}_{P(x)}\,y = \underbrace{2}_{Q(x)}$ é linear de 1ª ordem

2) $y' = \cos(x)\,y \Rightarrow y' \underbrace{-\cos(x)}_{P(x)}\,y = \underbrace{0}_{Q(x)}$ é linear

3) $y' = x^2 y^2$ não é linear, mas é separável.

$$\frac{1}{y^2} \cdot y' = x^2$$

Resolvendo:

fator integrante

$$y' + \frac{1}{x}\,y = 2 \overset{(\times x)}{\Longrightarrow} \underbrace{x y' + y}_{(x\cdot y)'} = 2x \Rightarrow (x \cdot y)' = 2x$$

$$\Rightarrow \int (x\cdot y)'\,dx = \int 2x\,dx \overset{(TFC)}{\Longrightarrow} x\cdot y + c_1 = x^2 + c_2$$

$$\Rightarrow x \cdot y = x^2 + C \Rightarrow y = x + \frac{C}{x}.$$

$$f(x) = F'(x) \Leftrightarrow \int f(x)\,dx = F(x)$$

$$y' + P(x)\,y = Q(x)$$

Fator integrante : $e^{\int P(x)\,dx}$

$$P(x) = \frac{1}{x} \Rightarrow e^{\int \frac{1}{x}dx} = e^{\ln x \cancel{+c}} = e^{\ln x} \cdot \cancel{e^{c}} = x \cdot \cancel{k}$$

$\underset{>0}{\underline{}}$

<u>Exemplos</u>: 1) $y' = \cos(x)\,y \Rightarrow y' \underbrace{-\cos x}_{P(x)}\, y = 0$

$$e^{\int -\cos x\,dx} = e^{-\operatorname{sen} x}$$

$$\therefore \left(y' - \cos x\, y\right) e^{-\operatorname{sen} x} = 0 \cdot e^{-\operatorname{sen} x} \Rightarrow e^{-\operatorname{sen} x} \cdot y' - \cos x \cdot e^{-\operatorname{sen} x} \cdot y = 0$$

$$\Rightarrow \left(e^{-\operatorname{sen} x} \cdot y\right)' = 0 \Rightarrow \int \left(e^{-\operatorname{sen} x} \cdot y\right)' dx = \int 0\, dx$$

$$\Rightarrow e^{-\operatorname{sen} x} \cdot y + C_1 = C_2 \Rightarrow e^{-\operatorname{sen} x} \cdot y = C \Rightarrow y = C \cdot e^{\operatorname{sen} x}.$$

2) $y' + 3x^2 y = 6x^2$

$$f(g(x))$$

$$e^x = \exp(x)$$

$$\ln x = \log_e x$$

$$\exp(\ln x) = x$$

$$\ln 2 = \boxed{\log_{\boxed{e}} 2 = \underline{\underline{x}} \iff \boxed{e}^{\underline{\underline{x}}} = 2}$$

$$e^{\ln x} = x \iff \ln y = x = \ln x$$

$$2^x = 8 = 2^3 \Rightarrow x = 3$$

$$2^x = 9 = 3^2 \Rightarrow x = \log_2 9$$

$$\ln(e^x) = x$$

$$\log_a x \qquad a^x$$